

Reg. No. :

Name :

I Semester M.Sc. Degree (CBCSS – OBE – Regular)

Examination, October 2023

(2023 Admission)

MATHEMATICS

MSMAT 01C02 : Linear Algebra

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. Let V be a finite-dimensional vector space over the field F and let $\{\alpha_1, \dots, \alpha_n\}$ be an ordered basis for V . Let W be a vector space over the same field F and let β_1, \dots, β_n be any vectors in W . Show that there is precisely one linear transformation T from V into W such that $T\alpha_j = \beta_j, j = 1, \dots, n$.
2. Show that every n -dimensional vector space over the field F is isomorphic to the space F^n .
3. If f is a non-zero linear functional on the vector space V , prove that the null space of f is a hyperspace in V .
4. Let V be a finite-dimensional vector space. Let W_1, \dots, W_k be sub spaces of V and let $W = W_1 + \dots + W_k$. Show that W_1, \dots, W_k are independent if and only if for each $j, 2 \leq j \leq k$, we have $W_j \cap (W_1 + \dots + W_{j-1}) = \{0\}$.
5. Define projection E of a vector space V . Show that $V = R \oplus N$, where R is the range and N is the null space of E .
6. Prove that an orthogonal set of non-zero vectors in an inner product space is linearly independent.



PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. Let T be the linear operator on C^2 defined by $T(x_1, x_2) = (x_1, 0)$. Let B be the standard basis for C^2 and let $B' = \{\alpha_1, \alpha_2\}$ be the ordered basis defined by $\alpha_1 = (1, i)$, $\alpha_2 = (-i, 2)$. What is the matrix of T relative to the pair B', B ?

8. Let F be a field and let f be the linear functional on F^2 defined by $f(x_1, x_2) = ax_1 + bx_2$. Find $T^t f$ if T is defined as $T(x_1, x_2) = (-x_2, x_1)$.

9. Let T be the linear operator on R^2 which is represented by the matrix

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$

Check whether T is diagonalizable or not.

10. Find the minimal polynomial for T represented in the standard ordered basis

by the matrix $\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$.

11. If $\{(3, 0, 4), (-1, 0, 7), (2, 9, 11)\}$ is a linearly independent set in R^3 , find an ortho normal basis for R^3 using Gram-Schmidt orthogonalization process.

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. a) Let V and W be vector spaces over the field F and let T be a linear transformation from V into W . If V is finite dimensional, prove that $\text{rank}(T) + \text{nullity}(T) = \dim V$.

b) Let T be a linear transformation from V into W . Show that T is non-singular if and only if T carries each linearly independent subset of V into a linearly independent subset of W .



13. a) Let V be a finite dimensional vector space over the field F and W be a subspace of V . Show that $\dim W + \dim W^\perp = \dim V$.
- b) Let T be a linear operator on the finite dimensional space V . Let c_1, \dots, c_k be the distinct characteristic values of T and let W_i be the characteristic vector space associated with the value c_i . If $W = W_1 + \dots + W_k$, prove that $\dim W = \dim W_1 + \dots + \dim W_k$.
14. a) State and prove Cayley – Hamilton theorem.
- b) Let V be a finite dimensional vector space over the field F and T be a linear operator on V . Prove that T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1) \dots (x - c_k)$ where c_1, \dots, c_k are distinct elements of F .
15. a) If $V = W_1 \oplus \dots \oplus W_k$, then prove that there exist k linear operators E_1, \dots, E_k on V such that
- i) each E_i is a projection
 - ii) $E_i E_j = 0$, if $i \neq j$
 - iii) $I = E_1 + \dots + E_k$
 - iv) the range of E_i is W_i .
- b) Let W be a finite dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W . Then prove that E is an idempotent linear transformation of V onto W , W^\perp is the null space of E and $V = W \oplus W^\perp$.
16. State and prove primary decomposition theorem.
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